

Analysis I – Week 1 review

Abstract

These points are intended to help you to review this week's contents; please take the time to do that, come up with examples and play around with the concepts. If you encounter problems, [take actions soon](#).

Whenever you think about something that you don't understand make a note so that you do not forget. You can discuss the questions with me, your small group tutor, and class-mates. Such question lists will help you to stay on top of things by asking pointed questions.

1. This week's review sheet is more philosophical since we have not too much material yet. **To review, reread the notes from page 22 to page 32.**
2. Thank you for your participation in class so far. If you keep this up, it will be more fun for us all. Don't be discouraged if there are points you miss in the lecture. You can simply not expect to understand 100% of the contents at once in 100% of the time; no matter how good you were in school. It is simply a fact that you need to get used to a certain way of thinking; give it time and withhold judgement.
3. We started off by discussion the very foundations of Analysis (and all of applied mathematics really). The real numbers. We have discussed that we do not prescribe the real numbers by writing down a bunch of numerals but by describing the properties that we would like them to have. We have discussed four axioms for addition (+), and four axioms for multiplication (\cdot).¹ The axioms of the real numbers, as we have them so far, can be clustered as

Field axioms: There are two operations (addition, multiplication) with $a + b \in \mathbb{R}$, $ab \in \mathbb{R}$ for any two $a, b \in \mathbb{R}$ which satisfy:		Field axioms
Axioms of addition	Axioms of multiplication	
Associative law	Associative law	
Commutative law	Commutative law	
Existence of the zero	Existence of the one ($\neq 0$)	
Existence of the negative	Existence of the inverse ($\neq 0$)	
Distributive law		

¹Please learn to make the dot not somewhere but where it belongs. It is $5 \cdot 3$ and not 5.3 .

4. You might want to check the axioms and the derived rules by some particular numbers. Like Let $a = 5$. Then $5^{-1} = \frac{1}{5}$ by the notation we introduced. We have $5 \cdot \frac{1}{5} = \frac{5}{5} = 1$ Further $(5^{-1})^{-1} = \frac{1}{\frac{1}{5}} = 5$ which illustrates $(a^{-1})^{-1} = a$ for $a \in \mathbb{R} \setminus \{0\}$. See also the Problem Sheet for the proof of Rules (I.12a)-(I.12c).
5. Remember the two mantras that we introduced.
- (a) I shall try to add a (helpful) 0.
 - (b) I shall try to multiply a (helpful) 1.

Thinking of Analysis as updating your brain with a cool new app that we call shall *Clear and economical thinking* (that rally includes also Liner Algebra, Mathematical thinking, etc...). Part of this app is a toolbox that you have to fill with a couple of tricks. So far we have two. In analogy, think of what a woodworker can do if she has a hammer. Now give her a saw, a plane². You get the idea.

6. Finally, to get a better handle on Axiomatic Thinking, I suggest you to read the speech of David Hilbert³ given on the matter; you can find it [here](#). It is not easy to read as there a some concepts you do not yet know, however, the point should become clear nevertheless. Also, you don't have to do it immediately. Know it is there. Settle in, let the first week sink in, and then get on.

You may find the Wikipedia article on [Axiomatic Systems](#) more accessible

²See [plane](#) in Wikipedia.

³German mathematician, 1862–1942.