Analysis I – Week 5 Review

1. Can you recall the definition of *convergence to* a for a given real sequence (a_n)? Write this down before you get to the next question.

2. Can you draw relevant pictures? You can have a look at this worksheet in GeoGebra too.

3. Do you understand, that

$$\forall a \in \mathbb{R} \quad \exists \varepsilon > 0 \quad \forall n_0 \in \mathbb{N} \ : \ \exists n \in \mathbb{N}, n \geq n_0, \ |a_n - a| \geq \varepsilon$$

   is the definition of divergence of a sequence (a_n).

4. Do you understand the interpretation that all but finitely many elements of the sequence must be in the interval $(a-\varepsilon, a+\varepsilon)$? Think. The definition says that for all $\varepsilon > 0$ there exists an index $n_0 \in \mathbb{N}$ such that $|a_n - a| < \varepsilon$ for all $n \geq n_0$. In the section on inequalities, we have learned that this means that

$$a_n \in (a-\varepsilon, a+\varepsilon) \quad \forall n \geq n_0.$$ [1]

   How many elements are (possibly) outside? Well, there are (at most) $n_0 - 1$ which is a finite number.

5. Can you write down the definition of a null-sequence?

6. In class we proved that $\left(\frac{1}{\sqrt{n}}\right)$ is a null sequence. With that one can prove that $\left(\frac{1}{n}\right)$ is a null-sequence. Can you do that. With that it follows that $\left(\frac{1}{n^k}\right)$ for all $k \in \mathbb{N}$ is a null-sequence. Why?

7. We have the notion of convergence and the notion of boundedness. We can expect that a convergent sequence should be bounded. Can you prove that? (Hint: (Reverse) Triangle inequality)

8. Do you remember some counterexamples to convergence? Are they reflecting everything that can go wrong?

9. Do you have a couple of counterexamples to reflect what it means not to converge to infinity.

10. Do you know how to construct a counterexample to the following: A sequence is increasing if $a_{n+3} \geq a_n$? What if the 3 is replaced by a 2 or a 5?

[1] We have that $|a_n - a| < \varepsilon$ is equivalent to $a - \varepsilon < a_n < a + \varepsilon$.