Analysis 2 – Week 10 Review

Abstract

These questions and remarks are intended to help you to review this week's contents; please take the time to do that, come up with examples and play around with the concepts. If you encounter problems, take actions soon. Work through your notes, ask me questions. Whenever you think about something that you do not understand make a note so that you do not forget. You can discuss the questions with me, your small group tutor, and class-mates. Such question lists will help you to stay on top of things by asking pointed questions.

This week's review remarks:

1. Recall the definition of pointwise convergence for a sequence (f_n) of functions $f_n : [a, b] \to \mathbb{R}$. Write down the entire! definition as you would in an exam.

2. Recall the definition of uniform convergence for a sequence (f_n) of functions $f_n : [a, b] \to \mathbb{R}$. Write down the entire! definition as you would in an exam.

3. Explain the difference between pointwise and uniform convergence. Give an example. Work out concretely what the difference is in your example.

4. Prove *uniform convergence* implies *pointwise convergence*.

5. Prove that *pointwise convergence* does not imply *uniform convergence* in general. (That means find a counterexample or recall one that you have been given.)

6. Prove that
$$f_n(x) = \frac{n}{nx + 1}$$
converges pointwise on $(0, 1)$. Determine the limit function. Do you notice something *strange*? Draw graphs of f_n.

7. Let (f_n) be a sequence of bounded functions $f_n : [a, b] \to \mathbb{R}$ and $f : [a, b] \to \mathbb{R}$. Suppose that $f_n \to f$ uniformly. Prove that f is bounded on $[a, b]$. (This is similar to Exercise 7.3. In fact, if you prove the assertion here, Exercise 7.3 becomes a corollary.)

8. Does 6 contradict 7?

9. Recall the definition of a step function. Write down the entire! definition as you would in an exam.

10. Recall the definition of the step-function integral. White it down and explain the notation.

11. State properties of the step-function integral and explain them with graphs; all of them!