Analysis 2 – Week 2 Review

Abstract

These questions and remarks are intended to help you to review this week's contents; please take the time to do that, come up with examples and play around with the concepts. If you encounter problems, take actions soon. Work through your notes, ask me questions. Whenever you think about something that you do not understand make a note so that you do not forget. You can discuss the questions with me, your small group tutor, and class-mates. Such question lists will help you to stay on top of things by asking pointed questions.

1. Watch the youtube clip When Pi is Not $3.14\ldots$ on the channel Infinite Series. It is also linked next to Problem Sheet 1.

2. Have you carefully read the lecture notes of this week? If not, you should start with that and then fp though this questions and remarks with paper and pencil.

3. As said in lectures, from now on, we will always use $\|x\|_2$ to denote the Euclidean length of a vector and not $|x|$. We use the symbol $|x|$ exclusively to mean the modulus of the real number x. In mathematical Methods, you will probably use $|x|$ to mean both, the modulus of a real number and the Euclidean length of a vector x.

4. We started this week's discussion by introducing two new notions, norms and metrics. They generalise the notions of length and distance.

   (a) Can you repeat the definition of a norm?

   (b) Can you repeat the definition of a metric?

5. Write down the definitions of $\|\cdot\|_p$ and ρ_p for $p \in [1, \infty]$. In relation to the previous question, what properties do $\|\cdot\|_p$ and ρ_p have?

6. Can you draw the unit spheres $\{x \in \mathbb{R}^2 : \|x\| = 1\}$ with respect to the norms $\|\cdot\|_\infty$, $\|\cdot\|_1$, and $\|\cdot\|_2$?
   Can you see how the picture is related to Theorem 1.2?
   To get started, calculate the norm of $x = [1\ ,\ 1]^T$ in all them norms and compare them. Look at a couple of vectors

7. In Analysis 2, we work with the norms $\|\cdot\|_p$ and the distances that they define by $\rho(ax, y) = \|x - y\|_p$. However, be aware that there exists metrics on $\mathbb{R}^d$ that do not come from norms, i.e. metrics are a more general concept.

8. Suppose $A \subseteq \mathbb{R}$. Then, the following are equivalent definitions of a bounded set.

   (B1) There exists a constant $C > 0$ such that $|a| \leq C$ for all $a \in A$.[1]

   (B2) There exists a constant $C > 0$ such that $|a - b| \leq C$ for all $a, b \in A$.

   Can you describe the conceptual differences between the two?
   Give related appropriate definitions for $A \subseteq \mathbb{R}^d$.

9. Have you done the reading of this week? If not, now is the time.

10. We ended this week by introducing the definition of an open set. Can you apply the definition to show that $(0, 1) \subseteq \mathbb{R}$ is an open set?

[1] This could be rephrased as follows: There exists a constant $C > 0$ such that $A \subseteq [-C, C]$.