Analysis 2 – Week 3 Review

Abstract

These questions and remarks are intended to help you to review this week's contents; please take the time to do that, come up with examples and play around with the concepts. If you encounter problems, take actions soon. Work through your notes, ask me questions. Whenever you think about something that you do not understand make a note so that you do not forget. You can discuss the questions with me, your small group tutor, and class-mates. Such question lists will help you to stay on top of things by asking pointed questions.

1. Compare your notes with the completed notes. Try the exercises on a blank sheet of paper before you copy them into your notes.

2. After your review, write down the definitions of the following terms. Write complete, comprehensible sentences.

   (a) Open ball $B_r(x)$,

   (b) Open set,

   (c) limit point,

   (d) isolated point,

   (e) closed set.

3. Can an open set have isolated points?
   Can a closed set have isolated points?
   Find examples if so in either case.

4. Consider the following sets and identify the limit points. What sets do you have to add to the given sets to make them closed? Sketch the sets.

   - $\left\{2 + \frac{1}{n} \;:\; n \in \mathbb{N}\right\}$,
   - $(0,1) \times (0,1)$,
   - $[0,1] \times [0,2)$.

   Write down complete arguments why the points you identified are limit points.

5. Can you freely switch between writing $\rho_2(x,y) < \varepsilon$ ($\|x-y\|_2 < \varepsilon$) and $y \in B_\varepsilon(x)$ respectively $x \in B_\varepsilon(y)$? Use open balls to formulate the definition of convergence of sequences $(a_n) \subseteq \mathbb{R}^d$.

6. The next questions relate to the reading. If you have trouble doing the exercises, always start with a simple example to think about the proof.

   - Consider $(0,1)$. Recall the proof that this interval is open. Now look at its complement $\mathbb{R} \setminus (0,1)$ and show it is closed. To which result does that relate? Could you prove the general result.
   - Consider A and B to be open. Can you prove that $A \cap B$, $A \cup B$ and $A \times B$ are then open as well? What about closed sets? You may wanna use the sketch on the next page.
   - Use the limit characterization of closed sets to show that $(0,1)$ is not closed.

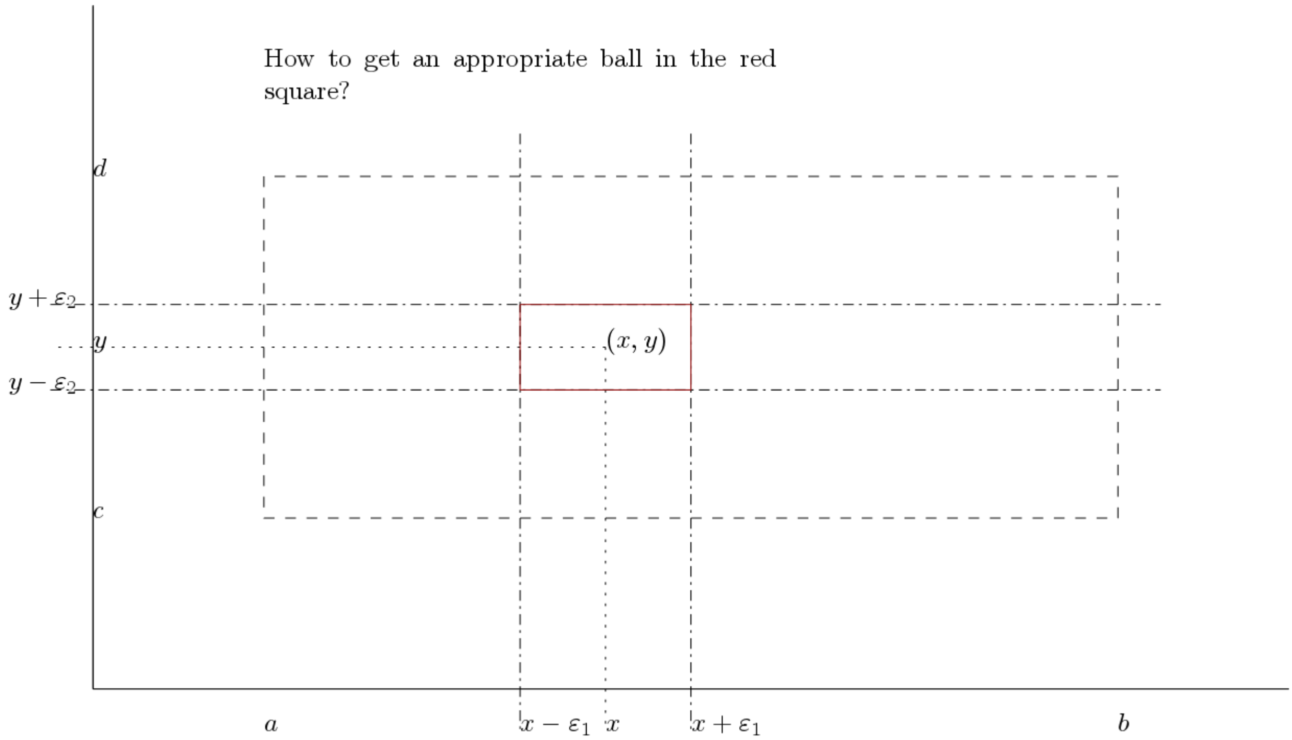

Figure 1: A sketch of the cross product of (a, b) and (c, d).