Analysis 2 – Week 6 Review

Abstract

These questions and remarks are intended to help you to review this week's contents; please take the time to do that, come up with examples and play around with the concepts. If you encounter problems, take actions soon. Work through your notes, ask me questions. Whenever you think about something that you do not understand make a note so that you do not forget. You can discuss the questions with me, your small group tutor, and class-mates. Such question lists will help you to stay on top of things by asking pointed questions.

Revision over the Easter break:

1. Evaluate your progress.[1]
2. Make a plan. See the time-management PDF attached to this sheet on LEARN.
3. If you have deficits in earlier chapters, it makes sense to work you way backwards. First work on an understanding of Chapters 3 and 4 for $d = m = 1$ (much of which then coincides with Mathematical Methods 1 and 2) and then use the contents of the previous chapters to grasp the notation and intuition needed for higher dimensions.
4. Use the many examples provided to illustrate the definitions and try to illustrate them with your own pictures.

This weeks review remarks:

1. This week, we introduced the notion of **continuity** for functions $f : \Omega \to \mathbb{R}^m$ for some $\Omega \subseteq \mathbb{R}^d$.
   (a) Can you recall the general definition.
   (b) Can you illustrate the general definition with a picture?
   With GeoGebra, you can investigate the (ε, δ)-definition.
   (c) Can you specialise[2] the definition to the following situations?
      i. $f : \mathbb{R} \to \mathbb{R}$,
      ii. $f : [-1, 1] \to \mathbb{R}^2$, and
      iii. $f : [0, 1]^2 \to \mathbb{R}^2$.
   (d) Under which condition is the following true:
      Let $\Omega \subseteq \mathbb{R}^d$, $f : \Omega \to \mathbb{R}^m$. Then, f is continuous at $x_0 \in \Omega$ if
      $$\lim_{x \to x_0} f(x) = f(x_0).$$
   (e) Can you explain what makes a function discontinuous?
      i. Can you give examples?
      ii. Can you write down the negation of the general definition of continuity?
2. Show that the following functions are continuous at the given points:

[1] How To Improve Your Metacognition and Why It Matters.
[2] That means choosing the correct ways of measuring the distances.

(a) Show that $f : [-5, +\infty) \to \mathbb{R}$, given by $f(x) = \sqrt{x+5}$, is continuous on $\mathbb{R}$.

(b) Show that $f : \mathbb{R}^2 \to \mathbb{R}^2$, given by

$$\mathbb{R}^2 \setminus \{0\} \ni \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mapsto \begin{bmatrix} \frac{x_1^2+2x_3+x_2}{(x_1^2+\frac{1}{2}x_2^2)^{\frac{1}{4}}} + 1 \\ x_1 + x_2 + 2 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \end{bmatrix} \mapsto \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$