Analysis 2 – Week 9 Review

Abstract

These questions and remarks are intended to help you to review this week's contents; please take the time to do that, come up with examples and play around with the concepts. If you encounter problems, take actions soon. Work through your notes, ask me questions. Whenever you think about something that you do not understand make a note so that you do not forget. You can discuss the questions with me, your small group tutor, and class-mates. Such question lists will help you to stay on top of things by asking pointed questions.

This week's review remarks:

1. Without going to the notes recall Fermat's, Rolle's and the Mean Value-Theorems. Before you check your results, draw also pictures of the situation of every theorem and work out the proofs which depend as Fermat $\Rightarrow$ Rolle $\Rightarrow$ Mean Value Theorem (MVT). Write the proofs in complete form not just scribbling half a line. Exercise writing down a comprehensive argument! This is what you have to do in the exam! Only then check back with the notes.

2. Do the exercises on Problem Sheet 5, Section 4. When you have done that successfully and understand what is going on, try your hands on the extra section.

3. We introduced the notion of pointwise convergence for a sequence of functions $(f_n) \subseteq C[a, b]$.[1] Recall the definition of pointwise convergence for a sequence $(f_n)_{n\in\mathbb{N}_0}$. Write down the entire! definition as you would in an exam.

4. An example was (f_n) given by $f_n : [-1, 1] \to \mathbb{R}$ with $f_n(x) = e^{-nx^2}$. We showed that this sequence converges pointwise to the function

$$\mathbb{R} \ni x \mapsto f(x) = \begin{cases} 1 & : \quad x = 0 \\ 0 & : \quad x \neq 0 \end{cases} .$$

   Do that again with the (ε, n_0)-definition of pointwise convergence given in the Lecture Notes. Find an explicit n_0 to illustrate the dependence on x and ε.

5. Do the possible problems on Problem Sheet 6.

[1] Remember that this means that for all $n \in \mathbb{N}$, we have that $f_n \in C[a, b]$. In other words, the sequence, considered as a set, is a sub-set of $C[a, b]$. Further note, that $C[a, b] = C([a, b])$.