

Subriemannian Geometry and Nilpotent Lie Groups

WOLFRAM BAUER

(Leibniz Universität, Hannover, Germany)

bauer@math.uni-hannover.de

Subriemannian manifolds model systems under non-holonomic constraints and their study vastly generalizes Riemannian geometry. Global and local aspects of such spaces and their interactions have attracted strong attention during the last years. Subriemannian structures and induces PDEs nowadays appear in many different areas of mathematics and physics as well as in applied sciences. Applications arise in control theory, hypoelliptic diffusion, geometric measure theory, potential analysis, optimal transport or harmonic analysis.

In analogy to the tangent space in Riemannian geometry, nilpotent Lie groups form the local "flat" models of a Subriemannian manifold. Their study forms the first step in the analysis and local approximation of "curved" Subriemannian spaces. Generalizing the Laplace-Beltrami operator in Riemannian geometry one considers the hypoelliptic sub-Laplacian Δ_{sub} on M . This operator and the induced heat equation on M and, in particular, on nilpotent Lie groups is our central object of interest. In fact, it provides links between analytic (e.g. spectral theoretical) and geometric quantities.

During five lectures we aim to shed some light on the role of nilpotent Lie groups G and hypoelliptic operators on G to the analysis of Subriemannian manifolds. "Sum-of-squares" of left-invariant differential operators on nilpotent Lie groups are interesting in their own right and can be applied to prove the existence of fundamental solutions of homogeneous Hörmander operators based on global lifting methods. Depending on time and interest of the participants we will discuss the following topics:

- (1) Introduction to Subriemannian geometry: basic properties and examples
- (2) Nilpotent approximation, Popp measure construction and intrinsic sub-Laplacian
- (3) Small time heat kernel expansions for degenerate operators
- (4) Folland's global lifting theorem and fundamental solution of degenerate operators
- (5) Local solvability for a class ultra-hyperbolic operators on pseudo- H -type groups