

Weyl Calculus on graded Lie groups

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In this mini-course we will introduce a pseudo-differential calculus on graded nilpotent Lie groups recently developed in collaboration with D. Rottensteiner and M. Ruzhansky. This calculus represents the Weyl calculus on the Heisenberg group and the right candidate Weyl calculus in the general graded group setting. This Weyl quantization, along with the corresponding calculus, will be suitably identified within the family of the so-called τ -quantizations.

Throughout the lectures we will cover the following arguments.

- Preliminaries on the necessary tools on nilpotent Lie groups: representations, Fourier transform, left/right invariant vector fields, Taylor expansion.
- Introduction to the Kohn-Nirenberg quantization on nilpotent groups introduced by M. Ruzhansky and V. Fischer.
- Singular integrals: the fundamental role of kernel estimates.
- τ -quantizations on graded groups and explicit examples in the case of the Heisenberg group.
- τ -calculi on graded Lie groups: asymptotic formulas.
- Invariance properties and the Weyl quantization.
- The Weyl quantization on the Heisenberg group and the candidate in the general graded group setting.

At times, to simplify the exposition, we will restrict the analysis to the case of the Heisenberg group.