Advanced Spatio-Spectral Analysis in Higher Dimensions

Azita Mayeli
City University New York (CUNY)
AMayeli@qcc.cuny.edu

These lecture series explore the theoretical foundations and applications of spatio-spectral limiting operators (SSLO) in higher dimensions, extending the study of prolate spheroidal wave functions and their role in time-frequency analysis. The lectures cover the eigenvalue distribution of these operators, providing insights into their behavior and implications in various fields such as signal processing, imaging, and data compression.

Within these lectures, we will focus on the following specific topics:

- **Introduction to Spatio-Spectral Concentration Problem**: An overview of the Heisenberg uncertainty principle and the role of prolate spheroidal wave functions in one-dimensional settings.
- **Introduction to Spatio-Spectral Limiting Operators (SSLO)**: Study of the operators and their spectral analysis.
- **Eigenvalue Distributions of SSLO**: Study of the eigenvalue distribution for SSLO, including asymptotic distribution and clustering behavior of eigenvalues.
- **Wave Packets and Orthonormal Bases**: Techniques for designing wave packets that serve as approximate eigenfunctions of spatio-spectral limiting operators, focusing on their application in higher dimensions.
- **Quantitative Bounds and Applications**: Analysis of quantitative bounds on eigenvalue distributions within given spatial and frequency domains, with a focus on signal manipulations.

Relevant reading: [1], [2], [3], [4], [4]

References.

[1] Arie Israel. *The Eigenvalue Distribution of Time-Frequency Localization Operators*. 2015. arXiv: 1502.04404 [math.CA].

[2] Arie Israel and Azita Mayeli. "On the eigenvalue distribution of spatio-spectral limiting operators in higher dimensions". In: *Appl. Comput. Harmon. Anal.* 70 (2024), Paper No. 101620, 28. DOI: 10.1016/j.acha.2023.101620.

[3] Santhosh Karnik, Justin Romberg, and Mark A. Davenport. “Improved bounds for the eigenvalues of prolate spheroidal wave functions and discrete prolate spheroidal sequences”. In: *Appl. Comput. Harmon. Anal.* 55 (2021), pp. 97–128. DOI: 10.1016/j.acha.2021.04.002.

[4] H. J. Landau and H. O. Pollak. “Prolate spheroidal wave functions, Fourier analysis and uncertainty. III. The dimension of the space of essentially time- and band-limited signals”. In: *Bell System Tech. J.* 41 (1962), pp. 1295–1336. DOI: 10.1002/j.1538-7305.1962.tb03279.x.