2.5 Learning Analysis

Section 2.5 forms this week's reading for Wednesday. It is modified from Chapter 4 of How to Think about Analysis (I have just adjusted a few lines to apply specifically to this module). Read all the subsections until you get to the next italicized comment telling you that you've reached the end.

2.5.1 The Analysis experience

Here is what happens when I teach Analysis. In week 1, everyone is in a good mood because they're starting something new. In weeks 2 and 3, there is a buildup of increasingly challenging material. In week 4, the mood in the lecture theatre is dreadful. The whole class has realized that this is difficult stuff and that it isn't going to get any easier. Everyone hates Analysis and, by extension, quite a few people hate me. I am not fazed by this, though, because I have taught Analysis about twenty times now and I know what will happen next. In week 5, everyone will feel slightly better, even if no-one can quite explain why. In week 7, a small number of people will approach me and tell me shyly that, although Analysis is challenging, they're starting to think they might like it. By the end of the course, these people will be telling anyone who will listen that Analysis is brilliant, and lots of other students will admit that now that they're getting the hang of it, they can see why people think it's a great subject.

The question for a new student, then, is how to handle it when the work gets difficult and you start to feel negative. Some students turn the negativity inwards: they lose confidence, experience self-doubt about their mathematical ability ('Perhaps I'm not good enough for this?'), and sometimes become withdrawn. Others turn it outwards, expressing frustration and anger about their lecturers ('He's a terrible teacher!') and sometimes, a bit nonsensically, about the mathematics itself ('I don't know why they're teaching us this rubbish—this isn't maths!'). These reactions both arise naturally when people feel a loss of control and consequently get defensive. But neither is very productive. So what is the alternative?

Well, most people do experience a bit of difficulty when first learning Analysis. This is just a fact of life. So, in my view, the trick is simply to expect this as a normal part of the learning experience, and ride it out. If you are ready for a bit of a challenge, you'll be better placed to handle the emotions without hiding away or acting out—you can say to yourself 'Well, okay, I was expecting this,' and continue to study in a sensible way, knowing that things will gradually come together. This week's reading is about practical approaches to doing that.

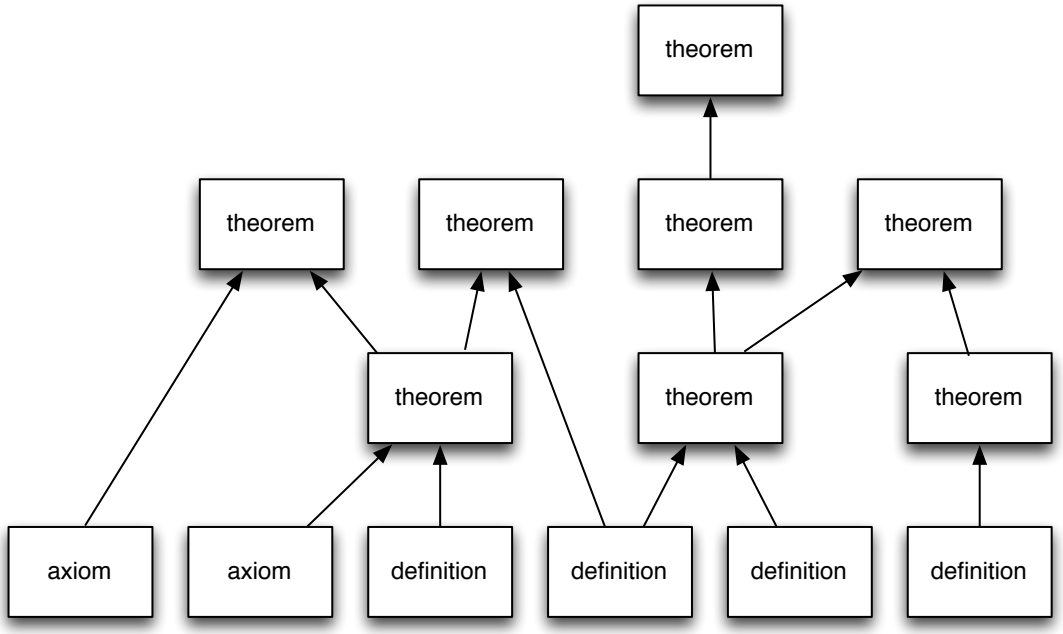
2.5.2 Keeping up

In Analysis, as in any undergraduate mathematics module, the big challenge is keeping up. If you're taking a decent degree course, this will be difficult. No-one is trying to teach you stuff that you will find easy—what would be the point of that? Also, you will be busy, with other courses and with the rest of your life. So it is very unlikely that you will be on top of everything all the time. You should try not to be distressed by this, because distress doesn't help—negative emotions just impede effective study. The thing to do is to accept that you will not always have perfect knowledge of everything, and work in an intelligent way that allows you to maintain *sufficient* knowledge of the *important* things.

When I say *sufficient* knowledge, I mean enough knowledge to give you a fighting chance of making sense of new material. By the time you are a few weeks into a module, you are unlikely to understand everything in every lecture—I certainly didn't when I was an undergraduate. But you want to have enough under your belt that you can follow the big sweep of the theory development and understand some of the details. When I say the *important* things, I mean the central concepts that come up again and again. At any given time, it is unlikely that you will be able to explain the nuances of every proof, but you want to know the main definitions and theorems so that you can recognize when and how they are used in new work. With that in mind, here is what I would prioritize.

First, you absolutely must know your definitions. In Analysis, it is sometimes tempting to be lax about this, because many of the words used ('increasing', 'convergent', 'limit' etc.) have everyday meanings, and because concepts in Analysis can often be represented using diagrams. Both of these things will tempt you into thinking that intuitive understanding is sufficient. *It isn't.* Definitions are central to any theory in advanced mathematics—they are key to understanding what is really meant by the theorems and what is going on in many of the proofs. If you think you understand the subject without knowing your definitions properly, you are kidding yourself. Because of this, I would start a definitions list now. Keep this on a piece of paper at the front of your folder (even if you keep most of your notes on an electronic device, I would still use paper for this). Every time you encounter a new definition, add it to the list. Study the list regularly, perhaps test yourself on it periodically, and be alert in lectures for defined words—every time lecturers use one, they mean it in exactly the sense captured by the definition.

Second, it is a good idea to be conversant with the main theorems. These capture relationships between concepts, so knowing what they say—even if you don't fully understand the proofs—will give you an overview of the module. Think thoroughly about theorem meanings—a few minutes spent relating a new theorem to examples or diagrams is likely to fix it in your mind. Also, you could get a sense of what theorems are coming by reading ahead. I would consider keeping a theorems list too. Indeed, I would go beyond list-making and construct a *concept map* (sometimes called a *mind map* or a *spider diagram*). Because of the way theory is built up, it often makes sense to use a diagram indicate which theorems (and definitions) are used to prove which other theorems. You could make a concept map that looks something like this, with the words in the boxes replaced by names or abbreviations for the specific definitions and theorems in this module:



Those are the things I would prioritize. If you find yourself getting behind or otherwise short of time, do those before you do anything else. Don't try to go back to where you last understood everything and work forwards from there—doing so will be ineffective, because the course will move on faster than you do, and you will end up in lectures where you do not understand anything. Analysis is hierarchical and thus unforgiving of students who do not keep up with the main building blocks. Prioritizing as suggested here will usually give you sufficient knowledge of the important things; it will allow you to identify key concepts and relationships in new lectures, and will provide you with a framework for more detailed study, as discussed below.

2.5.3 Avoiding time-wasting

Because keeping up is challenging, you do not want to waste any time. So it is worth thinking about how much study time you've got and how you're going to use it. This module involves three one-hour lectures per week. In such a system, I think it reasonable to spend a further three to four hours per week on independent study. If you do that for all your subjects, you'll probably end up with a standard 40-hour working week, which is about right.

Now, three to four hours isn't very much. You can tell yourself you're going to do more than that if you like, but most people don't, so it's probably more important to make the three to four hours count. During that time, you will have two things to do: study your lecture notes (or a textbook), and work on problems. I put the tasks in that order for a reason. In order to work effectively on the problems, you will need to be familiar with the material in your notes. If you are, you will find that many problems make you think 'Ah, we did something related to this on Wednesday.' If you aren't, you will waste a lot

of time having no idea how to start and staring into space. So, notes first.

I suggest spending perhaps 60-90 minutes studying your recent notes. This does not mean reading them without really thinking, though. Read everything carefully, following the study suggestions for definitions, theorems and proofs from the notes. Aim for good self-explanations (specific self-explanation training will be provided next week), but don't obsess over anything. 60-90 minutes isn't that long, and you want to be at least somewhat familiar with everything. So, if you have spent a few minutes thinking properly about something but you still don't really get it, get out a piece of paper, write 'Questions about Analysis' at the top, and make a note of where this thing is and what exactly you don't understand. Be precise—sometimes nailing down the problem allows you to sort it out and, if it doesn't, you will have a specific note to come back to so you don't lose the thinking you have already done.

Once you've studied your notes, begin work on the problems. Depending on how you divide up your time, you'll have between two and three hours for this. That will not give you very much time for any given problem, so again you don't want to waste any. Because of this, I suggest a first pass in which you spend perhaps ten minutes on each problem. Some problems you will be able to finish in this time, especially if they involve routine warm-up exercises or direct applications of an idea that you've just studied. (In such cases, see how much you can do without looking at your notes—this might take slightly longer but, if you can construct or reconstruct something for yourself, you will remember it better in the long run.) Other problems you will not be able to finish in ten minutes. If you are making good progress, you might want to carry on for a bit longer. If you get stuck, try some or all of these things to get unstuck:

- Work out exactly what the problem is asking for—can you explain this out loud?
- Look up the definitions of any relevant mathematical words, and write what you know and what you want to show in terms of these definitions.
- Try drawing a diagram or thinking about a specific example.
- Study a relevant bit of your notes. Don't just look for things to copy, though—that won't work in Analysis. Instead, spend a bit of time trying to understand the notes and thinking about how similar ideas might apply for your problem.
- Try to write down exactly what is making the problem difficult. Often, the attempt to do this will help you sort it out.

If you've tried those and you're still stuck, make a note on your 'Questions about Analysis' sheet and move on—those other problems are still waiting.

Now, I said 'at a first pass' because I think problem solving in Analysis should be a multiple-pass task. You want to have a go, then have a break for a day or two, then have another go. Magical things will sometimes happen in the break—your brain will make new connections and you'll see new ways forward. So you probably want to break up

your study into at least a couple of blocks. Indeed, you should do that anyway, because thoughtful study is intellectually effortful—if you decide to spend four hours at a stretch studying Analysis, I guarantee that you will waste the last two simply because you will run out of energy.

2.5.4 Getting your questions answered

Next, what to do with your 'Questions about Analysis' list? For a start, keep an eye on it. Sometimes, working on problems will make you think about an idea in a different way, and you'll be able to cross off something that you added when studying your notes. Sometimes, when you've had a break for a couple of days, a quick re-read of your notes will make something click, and you'll be able to finish a problem and cross that off too. After that, here's what I'd do.

First, get together with a friend or two and work systematically through your respective lists. Everyone thinks a bit differently, so you will probably be able to fill some gaps for one another. Doing this will also force you to speak about Analysis, helping you to become fluent in talking about the concepts and explaining your arguments. Fluency is important, so don't worry if you trip over your words at first. Just have another go—you will only get more confident with practice. Sharing ideas will also help you to become a good mathematical listener. Pay close attention to what your friends are saying and, if you are not sure you understand, say so, and try to specify what is confusing you. Doing this will help your friends to articulate their thoughts more clearly. Again, this is a valuable skill that will help all of you to speak more confidently to lecturers and other tutors. Of course, as in individual work, don't get obsessed—if you can't sort something out between you in a reasonable amount of time, perhaps your effort would be better spent elsewhere.

Once you've shared your knowledge with friends, take your remaining questions to an expert (you can always go to the expert first, of course, but consider the issues about developing communication skills): perhaps your tutor, perhaps me, perhaps one of the Mathematics Learning Support Centres. Whoever you see, take your list and your problem sheets and all your relevant notes, and make sure that your list has page or section or question numbers on it—you want to be able to find everything with minimal fuss. If seeing someone involves arranging a specific meeting, consider asking whether you and your friends can go together—that should make the process more efficient. And do not be shy about asking questions, even if you have a long list. Trust me, a student asking specific questions from a well-organized list is always impressive.

Taking this approach should mean that most of your questions are answered most of the time. However, do be realistic. Following this advice will still leave you with gaps. Sometimes there will not be time to sort everything out. Sometimes there will be time to sort everything out, but two weeks later you will realize that you've now forgotten why something works and you need to think it through again. It should be possible to minimize that problem by making decent notes—when you've overcome confusion about

something, recording how you changed your thinking will facilitate quick review. Overall, if you get yourself organized at approximately the level suggested here, you will keep up with the main ideas, you will understand at least some of each new lecture, and you will develop a solid block of knowledge that you can build on when you start preparing for tests and exams.

2.5.5 Adjusting your strategy

In this chapter I have suggested a specific way to organize your studies. I should say that I don't really expect anyone to behave in precisely this way. You will be subject to constraints about when you can study, to personal preferences about your work habits, and to the shifting requirements of other aspects of your academic work and your social life. So you should reflect occasionally on how things are going, and be ready to adjust. If you need longer to study your notes, adjust your timings; if you need time to study for a test in another subject, cut back to the essentials in Analysis for a week; if one of your friends is great for social outings but a bit rubbish at concentrating on Analysis, quietly make alternative or extra arrangements for discussions with others. And of course, if you're really into a problem, stare into space and think about it for hours, if you like. The advice here should be thought of as a useful place to start, and as a way to develop a routine that will keep you going through the challenging weeks.

You should now:

- Review the rest of this week's material in order to prepare for the activities on Wednesday.
- Apply the advice you just read. For example, get out your timetable, work out when you will fit in your Analysis study, and agree a time to with some friends to meet up and share answers.
- Visit `www.lboro.ac.uk/mlsc` to find out about the Mathematics Learning Support Centres and consider doing some of your study in one of them.
- Read and perhaps start work on this week's problems.

That concludes this week's reading.