

## How to study for Analysis

- **Read carefully and deliberately.** As you know, the way one should read in mathematics is quite different from how you may read a history book or a magazine or newspaper. In mathematics you must read slowly, absorbing each phrase. In the first semester, you should all have received a self-explanation training booklet which you should not forget. If you have lost your copy or have never received one, you can contact me and I will bring some to the lecture.
- **Think with pencil and scratch paper.** Working on mathematics you should always have a pencil and some sheets of paper ready and use them. Test out what is written in the lecture notes, construct examples and counterexamples, draw pictures.  
This will help to clinch the ideas and procedures in your mind before starting the exercises. After you have read and reread a problem carefully, if you still do not see what to do, do not just sit and look at it. Get your pencil going on scratch paper and try to dig it out. Try this "algorithm":
  1. Write down what you want to show.
  2. Write down what you know.
  3. What can you immediately deduce from the known? Apply some known simple inequalities and identities and see what you can get from it.
  4. Think of a strategy. And try to implement it.
  5. Have you used all information? Do you have numbers, vectors or functions. Have you used their specific properties?

Another good reason for getting things down is that it is much more easy to help you. Maybe you just overlooked something and with a small hint from me or tutors, you have a light-bulb experience. You should not rob yourself of that by asking to much of the solutions at once.

- **Be independent.** To be clear, being independent does not mean that you should ask no questions at all but to have good judgement over what to ask and when. Sometimes little things will cause considerable confusion or you do not know where even to begin with your studies. Then you should ask for help. Do not be afraid that your question may sound *dumb*. The only *dumb* action is to fail to ask about a topic that you have really tried to grasp and still do not understand. Some people seek help too soon and some wait too long. You will have to use good common sense in this matter.

- **Persevere.** Do not become frustrated if a topic or problem may completely baffle you at first. *Stick with it!* An extremely interesting characteristic of learning mathematics is that at one moment the learner may feel totally at a loss, and then suddenly have a burst of insight that enables her to understand the situation perfectly. If you don't seem to be making any progress after working on a problem for some time, put it aside and attack it again later. Many times you will then see the solution immediately even though you have not been consciously thinking about the problem in the meantime. There is a tremendous sense of satisfaction in having been persistent enough and creative enough to independently solve a problem that had given you a great deal of trouble.
- **Take time to reflect** To learn mathematics well you must take time to do some reflective thinking about the material covered during the last few days or weeks. It takes time for some ideas in mathematics to *soak in*. You may have to live with them a while and do reflective thinking about them before they become a part of you.
- **Concentrate on fundamentals.** Do not try to learn mathematics by memorizing illustrative examples.<sup>1</sup> You will soon become overwhelmed by this approach, and the further you go the less successful you will be. All mathematics is based on a few fundamental principles and definitions. Some of these must be memorized. But if you concentrate on these fundamentals and try to see how each new topic is just a reapplication of them, very little additional memorization will be necessary.<sup>2</sup>
- **Use Heuristics.**<sup>3</sup> If you work on a problem that you can not solve immediately try to consider a problem that is similar but somewhat easier. It is not always easy to find such problems but it will come to you with practice. For example, if you consider a function that depends on a parameter, you can study it by setting the parameter to a specific value. Then do the calculations and, after reaching a satisfying result, follow your calculations step by step with the general parameter. Many more methods may be found in [5] and [3]. Some historical points and further references can be found in [4]. Some evidence of student's benefits from following heuristic methods as well as further references can be found in [2].

---

<sup>1</sup>Which does not mean you should not have a couple of examples/counterexamples up your sleeve for any concept learned.

<sup>2</sup>For example: If you fully understood how one searches for extrema when one considers functions of one variable, only few additional things have to be memorized in the multi-variable case. (Even less with further knowledge in multi-variable calculus.)

<sup>3</sup>Heuristics is the study of means and methods of problem solving.

## References

- [1] Robert A. Bjork John Dunlosky, and Nate Kornell *Self-Regulated Learning: Beliefs, Techniques, and Illusions*, Annual Review of Psychology, 64(1) (2013). doi:[10.1146/annurev-psych-113011-143823](https://doi.org/10.1146/annurev-psych-113011-143823).
- [2] Gerald A. Goldin *Problem Solving Heuristics, Affect, and Discrete Mathematics: A Representational Discussion* in Theories of Mathematical Education, Springer, New York (2010). doi:[10.1007/978-3-642-00742-2\\_24](https://doi.org/10.1007/978-3-642-00742-2_24)
- [3] Zbigniew Michalewicz and David B. Fogel *How to Solve It: Modern Heuristics*, Springer, New York, 2nd edition, 2004. ISBN 9783540224945
- [4] Nicholas Mousoulides and Bharath Sriraman *Heuristics in Mathematics Education* in Encyclopedia of Mathematics Education, Springer, New York, pp. 253–255 (2014). doi:[10.1007/978-94-007-4978-8\\_172](https://doi.org/10.1007/978-94-007-4978-8_172)
- [5] George Polya *How to Solve It*, Princeton University Press, Princeton, Notable Centenary Edition, 2014 (original 1945). ISBN 9780691164076
- [6] Karl Randall *Improving study habits in mathematics*, The mathematics teacher, 55(7) (1962). [Jstor](#).