Sample questions similar to the final exam

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Question 1 *Let the function*

$$f_a(x) = \frac{x^2 - 4}{x^2 - a^2}$$

be given and $a \in \mathbb{R}_{>2}$ a parameter.

(a) Find the natural domain of f_a, the zeros, and determine the symmetry type.

(b) Find points of discontinuity and classify them.

(c) Find the monotonicity intervals of f_a and determine the position and type of the local extrema of f_a.

Remark 1 *Do the same as in Question 1 again for the function*

$$f_a(x) = \frac{9 - x^2}{x^2 - a^2}$$

with $a \in \mathbb{R}_{>3}$.

Question 2 *Solve the following questions. Justify your answers; calculations must be complete in the sense that one*[1] *must understand the reasoning.*

(a) Find a simple example of a function with three inflexion points.

(b) Find an example for a 1-periodic function.

(c) Let the function

$$f(x) = (1 + ax)e^{-ax}$$

be given.

(i) Find the coordinates[2] *of the local extrema of f and determine their nature.*

(ii) Investigate the monotonicity behavior of f.

[1] and can see that you know what you are doing

[2] The word coordinates implies that we are looking for x_0 as well as $f(x_0)$.

Question 3 *Consider the function*

$$f(x,y) = \frac{x^2-1}{y}.$$

Answer the following questions, justify your answers.

(a) Find the natural domain of f.

(b) Sketch the domain on which $f(x,y) \geq 0$.

(c) Compute the level curves for $c = -1$ and $c = 1$ and put them in your sketch.

Question 4 *Let the function*

$$f(x,y) = \ln(2x) - xy + 3y + \frac{2}{y}$$

be give.

(a) Find and sketch the natural domain of f.

(b) Compute the directional derivative in direction

$$a = \frac{1}{5}\begin{bmatrix} 4 \\ -3 \end{bmatrix}.$$

(c) Investigate f for stationary points and determine their nature.

Question 5 *Consider*

$$f(x,y) = e^{-xy} + x^3 - y^2$$

and let $x_0 = (1,0)$ be given.

(a) Compute the slope of f in the direction of $a = \left(\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\right)$ and in the direction of $\nabla f(x_0)$.

(b) Find the equation of the tangential plane in x_0 on the graph of f.

(c) Compute the slope of the level curve $f(x,y) = 0$ in the intersection points with the y-axis.